ME 243

Mechanics of Solids

Lecture 6: Stresses in Beams

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Assumptions in deriving equations of stresses

- The plane sections of the beam remains plane.
- The material in the beam is homogeneous and obeys Hooke’s law.
- The modii of elasticity for tension and compression are equal.
- The beam is initially straight and of constant cross section.
- The plane of loading must contain a principle axis of the beam cross section and the loads must be perpendicular to the longitudinal axis of the beam.

Flexure formula

- The stresses caused by the bending moment are known as bending or flexure stresses, and the relation between these stresses and the bending moment is expressed by the flexure formula.

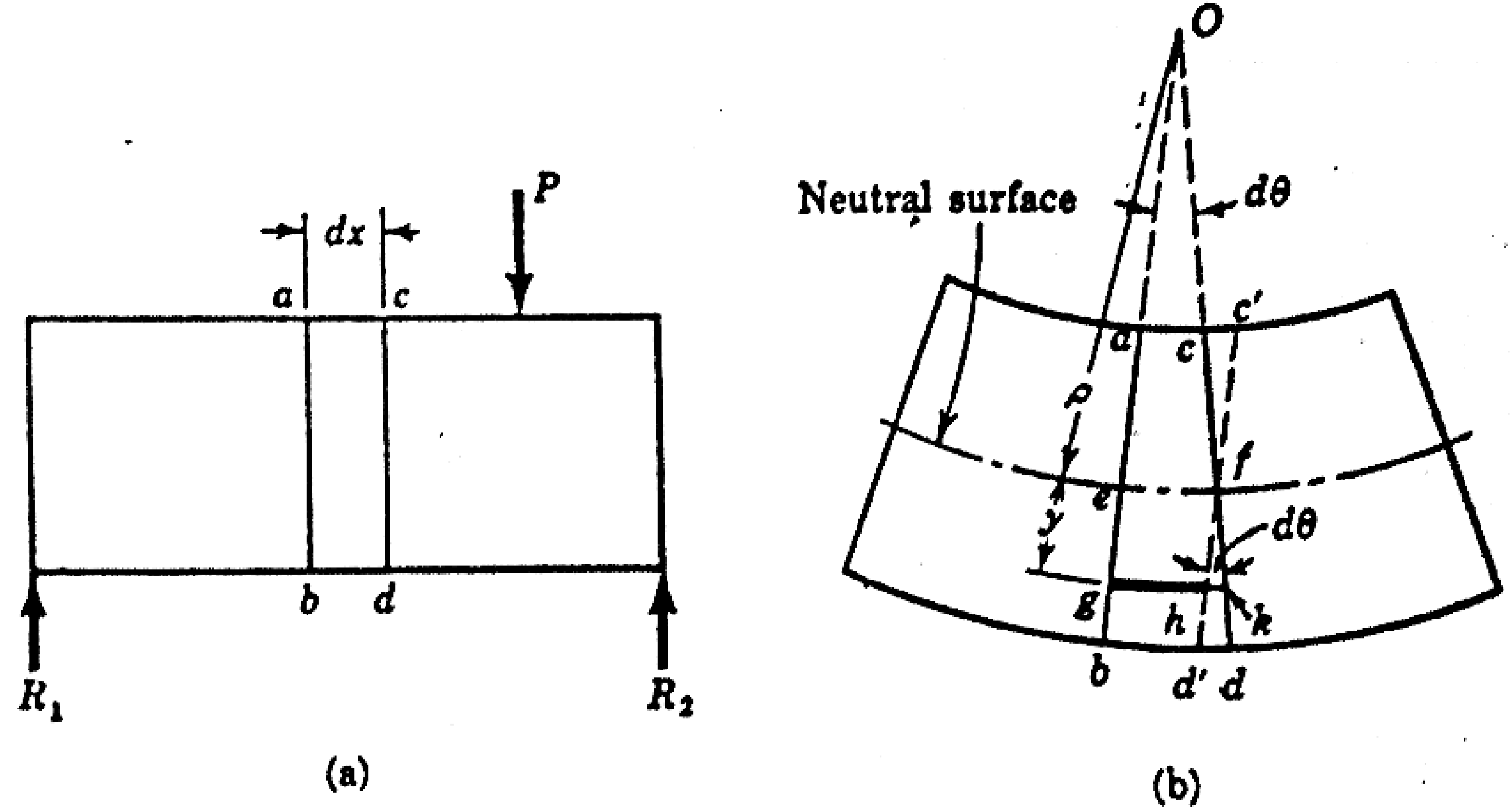
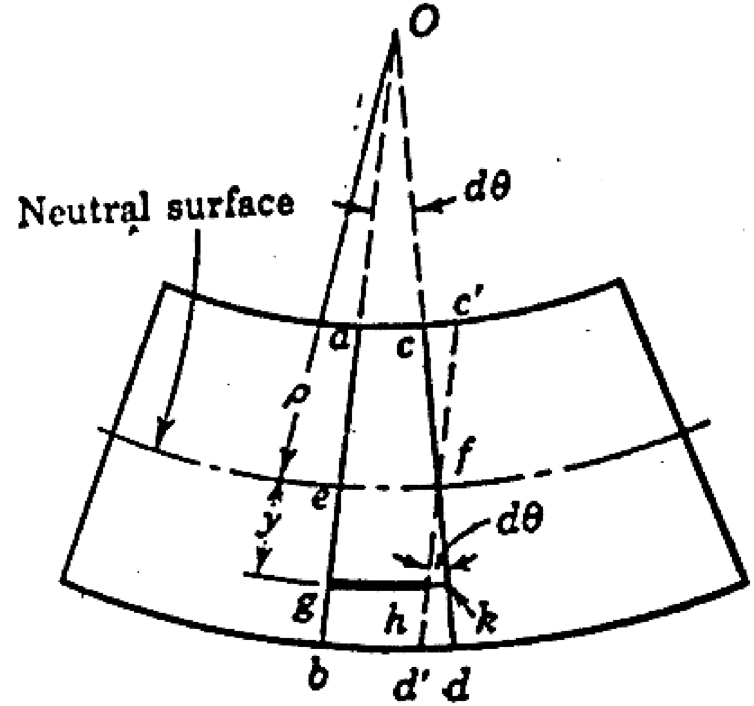


Figure (a) shows two adjacent sections, *ab* and *cd*, separated by the distance dx. Because of the bending caused by load P, sections *ab* and *cd* rotate relative to each other by the amount $d\theta$, as shown in Fig. (b) but remain straight and undistorted



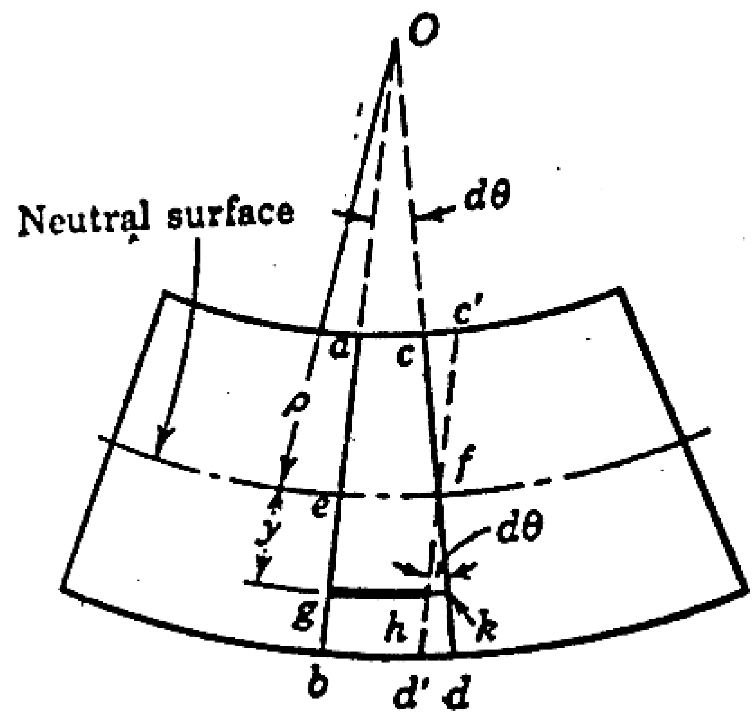
Fiber *ac* at the top is shortened, and fiber *bd* at the bottom is lengthened. Somewhere between them is located fiber *ef*, whose length is unchanged. Drawing the line *c'd'* through *f* parallel to *ab* shows that fiber *ac* is shortened an amount *cc'* and is in compression, and that fiber *bd* is lengthened by an amount *d'd* and is in tension.

The plane containing fibers like *ef* is called the *neutral surface* because such fibers remain unchanged in length and hence carry no stress.

Consider now the deformation of a typical fiber gh located y units from the neutral surface. Its elongation hk is the arc of a circle of radius y subtended by the angle $d\theta$ and is given by

$$\delta = hk = y\, d\theta$$

$$\text{Strain}, \varepsilon = \frac{\delta}{L} = \frac{y\, d\theta}{ef}$$



The stress in fibre gh is given by,

$$\sigma = E\epsilon = \left(\frac{E}{\rho}\right) y \quad \cdots (1)$$

To complete the derivation of the flexure formula, we apply the conditions of equilibrium. the external loads that act to one side o. an exploratory section are balanced by the resisting shear V_r and the resisting moment M_r. To create this balance, a typical element in the exploratory section is subjected to the forces shown in the pictorial sketch* in Fig. (c). The line of intersection between the neutral surface and the transverse exploratory section is called the *neutral axis*, abbreviated NA.

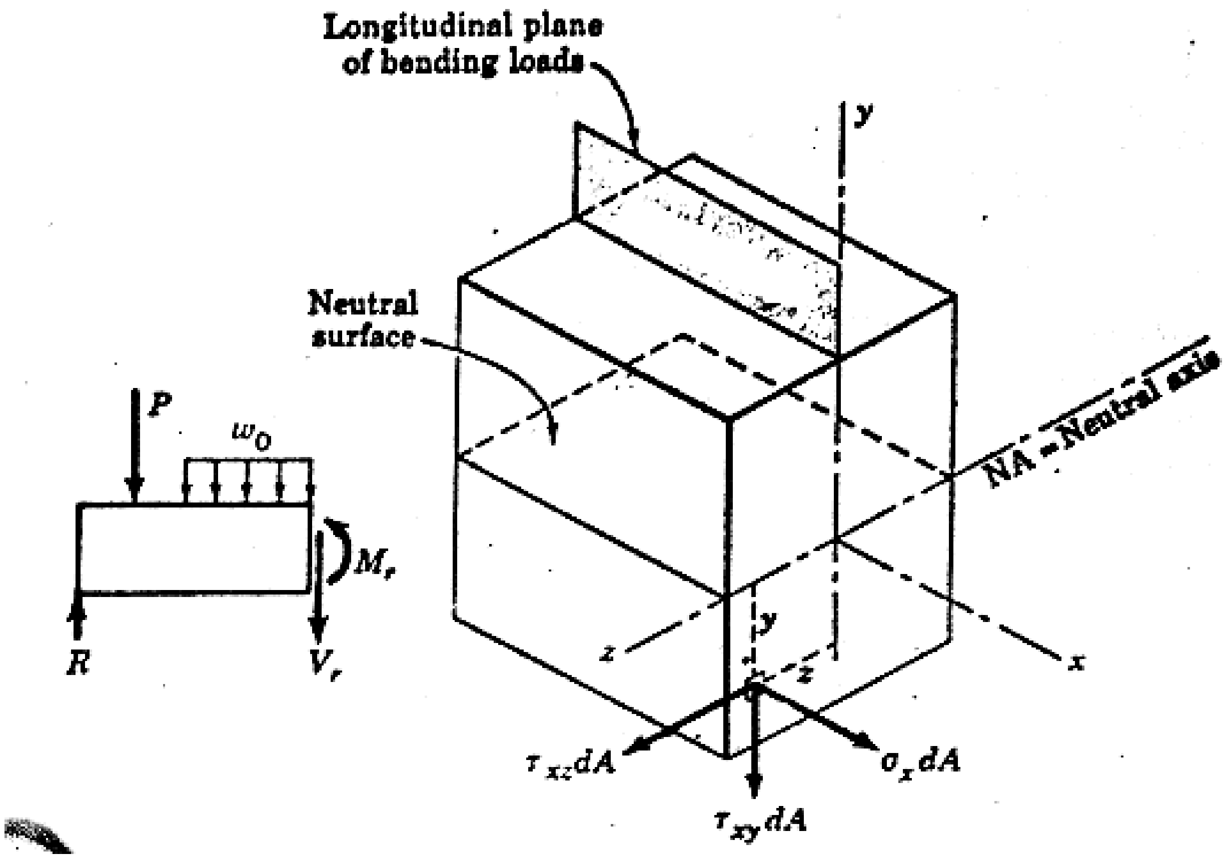


Fig. (c)

$$\sum F_x = 0 \Rightarrow \int \sigma_x \, dA = 0 \quad \cdots (2)$$

where, σ_x is equivalent to σ in eq^n (1).

$\therefore$ From ① & ②,

$$\frac{E}{\rho} \int y \, dA = 0$$

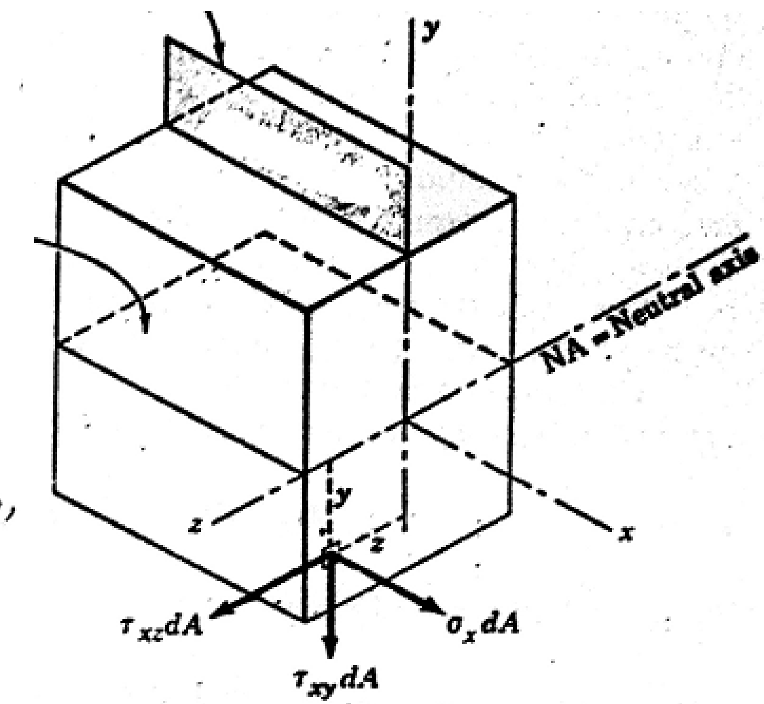
The constant ratio E/ρ is written outside the integral sign. Since $y\ dA$ is the moment of the differential area dA about the neutral axis, the integral $\int y\ dA$ is the total moment of area. Hence

$$\frac{E}{\rho} A\bar{y} = 0$$

However, since only y in this relation can be zero, we conclude that the distance from the neutral axis (which is the reference axis) to the centroid of the cross-sectional area must be zero; that is, *the neutral axis must contain the centroid of the cross-sectional area.*

Now,
$$\Sigma F_y = 0$$
$$\Rightarrow V = V_r \,,$$

Here, the resisting shear V_r is the summation of the shearing forces, $\tau_{xy} \cdot dA$, i.e, $V_r = \int \tau_{xy}\, dA$



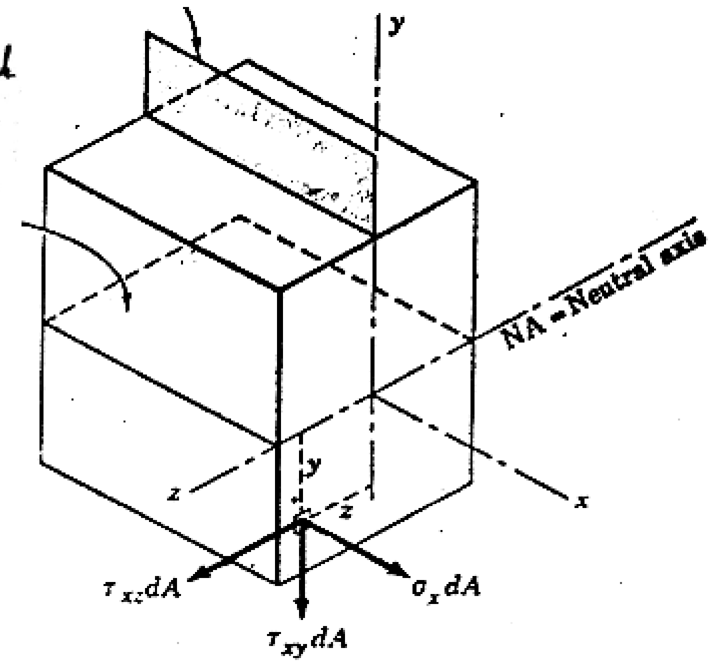
Again, $\Sigma F_z = 0$
$$\Rightarrow \int \tau_{xz}\, dA = 0$$

Since loading has no z- components, the system of shear forces $\tau_{xy}\, dA$ must be self-balancing.

Now, $\Sigma M_x = 0$

$$\Rightarrow \int y(\tau_{xy}\, dA) - \int z(\tau_{xy}\, dA) = 0$$

This condition is satisfied automatica-lly for sections that are symmetrical about y-axis.



Now, $\Sigma M_y = 0$

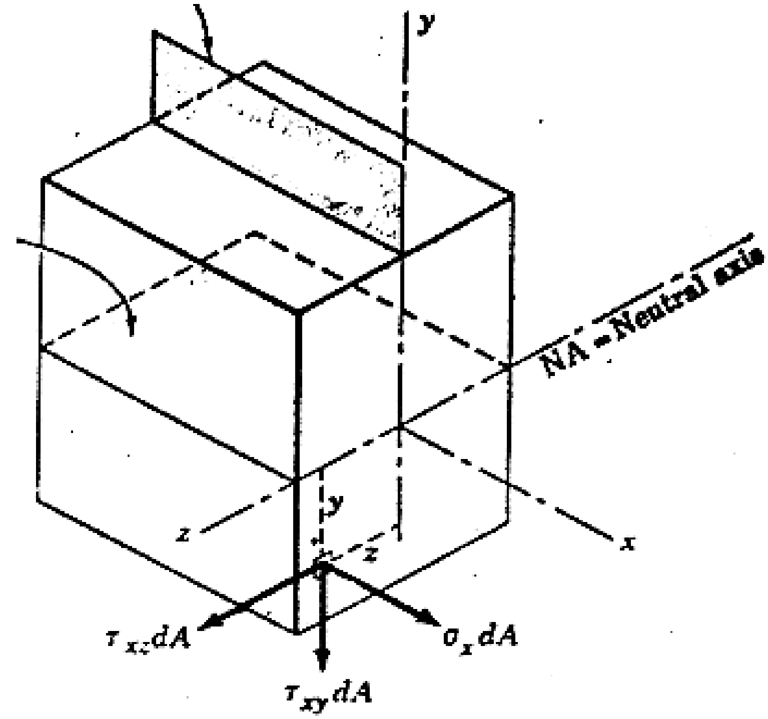
$$\Rightarrow \int z(\sigma_x\, dA) = 0$$

$$\Rightarrow \frac{E}{\rho} \int zy\, dA = 0$$

Here, $\int zy\, dA = P_{zy}$ = Product of inertia
= zero only if y or z is an axis of symmetry.

Now, $\Sigma M_z = 0$

$$\Rightarrow M = M_R$$



But, $M_R = \int y(\sigma_x \, dA)$

$$\therefore M = \int y(\sigma_x \, dA)$$

$$= \frac{E}{\rho}\int y^2 \, dA$$

$$= \frac{E}{\rho} I \quad \Rightarrow \quad M = \frac{EI}{\rho}$$

Here, 'I' is the moment of inertia of the area about the neutral axis.

$$\therefore \quad M = \frac{EI}{\rho}$$

$$\Rightarrow \quad \frac{1}{\rho} = \frac{M}{EI}$$

Here, ρ – radius of curvature

$\therefore \frac{1}{\rho}$ – curvature

$\therefore$ Curvature is proportional to bending moment

But ① $\Rightarrow$ $\frac{E}{\rho} = \frac{\sigma}{y}$

$$\therefore \quad \frac{E}{\rho} = \frac{\sigma}{y} = \frac{M}{I}$$

$$\Rightarrow \quad \boxed{\sigma = \frac{My}{I}}$$

Flexure stress

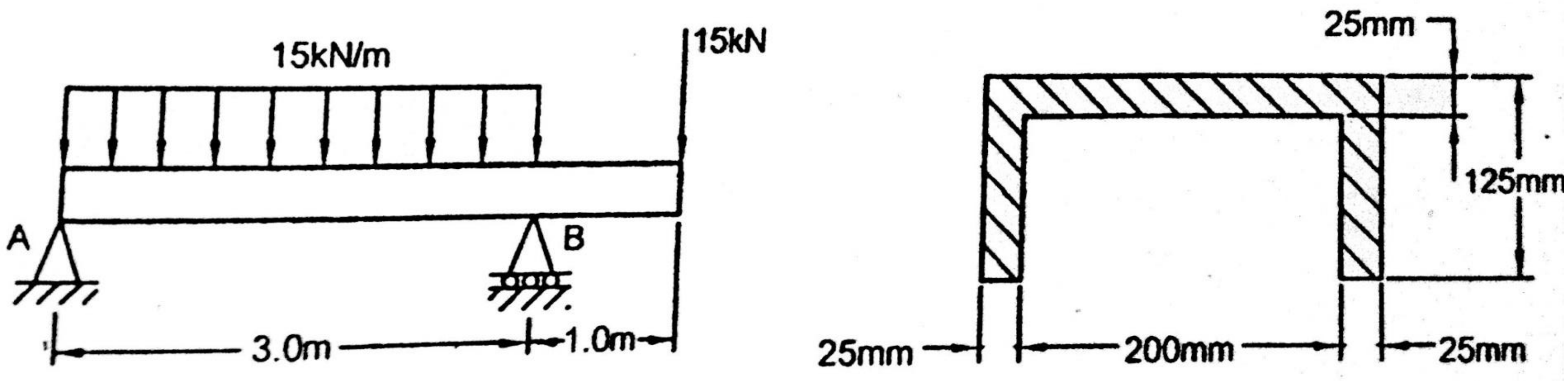
- The flexure stress in any section varies directly with the distance of the section form the neutral axis.
- Usually, ‘y’ is replaced by ‘c’, which is the distance from the neutral axis to the remotest element.
- The maximum flexure stress in any section is given by,

$$\text{Max. } \sigma = \frac{Mc}{I}$$

- Flexural stresses are maximum at top and bottom fibers.

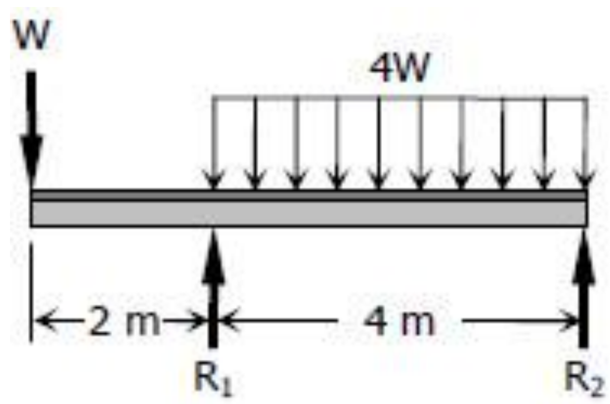
Problem#4.26(quamrul)

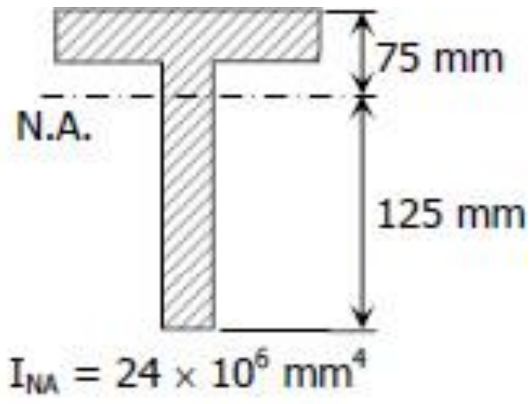
- An overhanging beam of channel section supports a uniformly distributed load of 15 kN at its free end. Determine the maximum tensile and compressive stresses developed in the beam.



Problem# 555 (singer)

- A beam carries a concentrated load W and a total uniformly distributed load of 4W as shown in Fig. What safe value of W can be applied if $\sigma_c \leq 100$ MPa and $\sigma_t \leq 60$ MPa?





Horizontal shear stress of beams

Consider two adjacent sections, (1) and (2), in a beam separated by the distance dx, as shown in Fig. 1, and let the shaded part between them be isolated as a free body. Figure 2 is a pictorial representation of this part, the beam from which it is taken being shown in dashed outline.

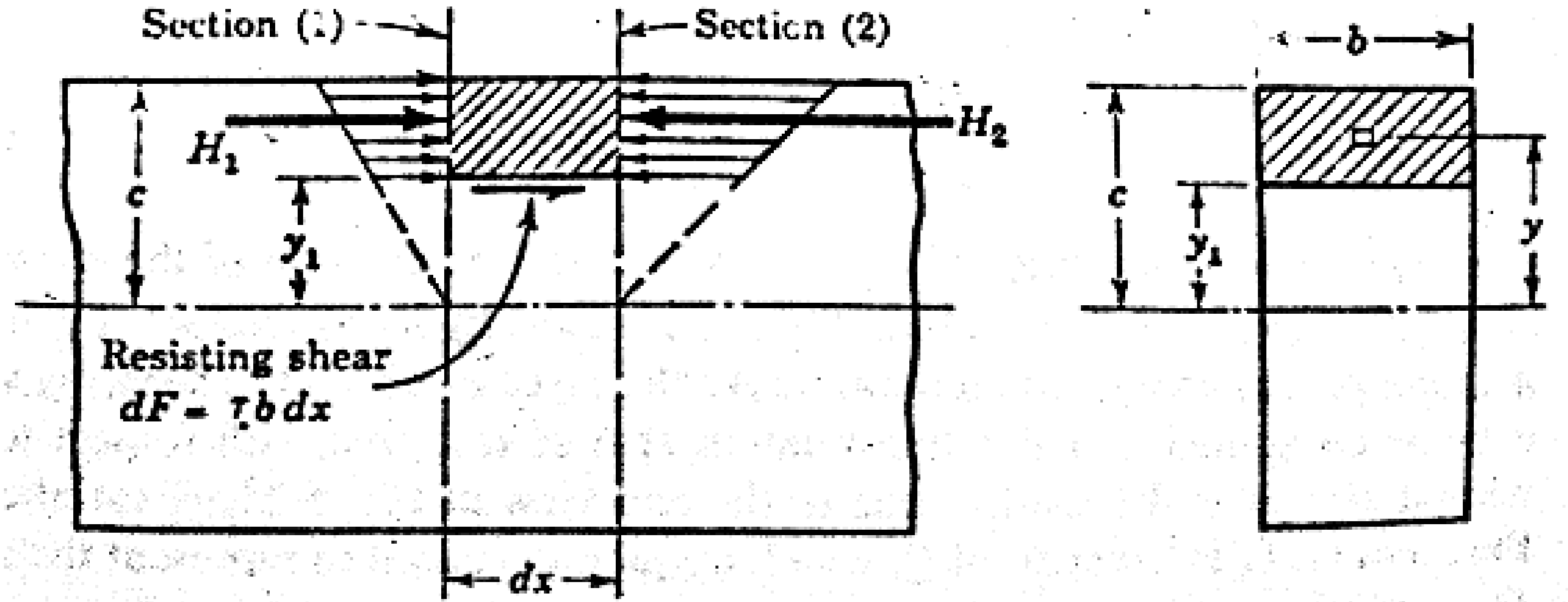
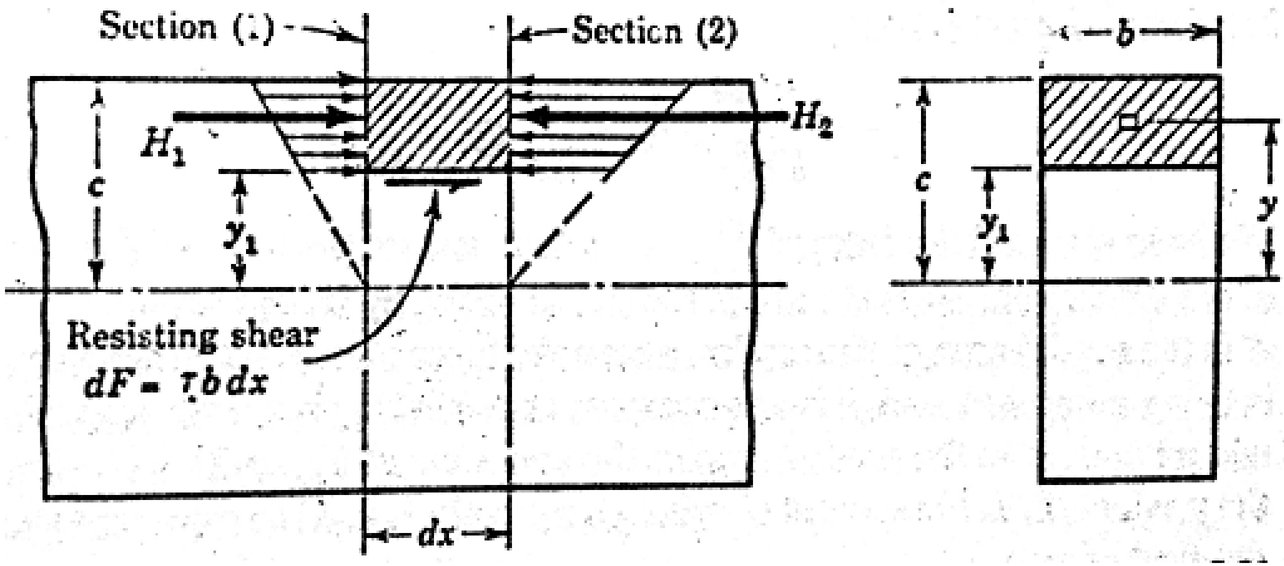


Fig : 1

Horizontal shear stress of beams

Assume the bending moment at section (2) to be larger than that at section (1), thus causing larger flexural stresses on section (2) than on section (1). Therefore the resultant horizontal thrust H_2 caused by the compressive forces on section (2) will be greater than the resultant horizontal thrust H_1 on section (1). This difference between H_2 and H_1 can be balanced only by the resisting shear force dF acting on the bottom face of the free body, since no external force acts on the top or side faces of the free body.



Horizontal shear stress of beams

Since $H_2 - H_1$ is the summation of the differences in thrusts $\sigma_2\, dA$ and $\sigma_1\, dA$ on the ends of all elements contained in the part shown in Fig. 1, a horizontal summation of forces gives

$$\Sigma F_H = 0$$

$$\Rightarrow dF = H_2 - H_1$$

$$= \int_{y_1}^{c} \sigma_2\, dA - \int_{y_1}^{c} \sigma_1\, dA$$

Putting $\sigma = \frac{My}{I}$, we get,

$$dF = \frac{M_2}{I}\int_{y_1}^{c} y\, dA - \frac{M_1}{I}\int_{y_1}^{c} y\, dA$$

$$= \frac{M_2 - M_1}{I}\int_{y_1}^{c} y\, dA$$

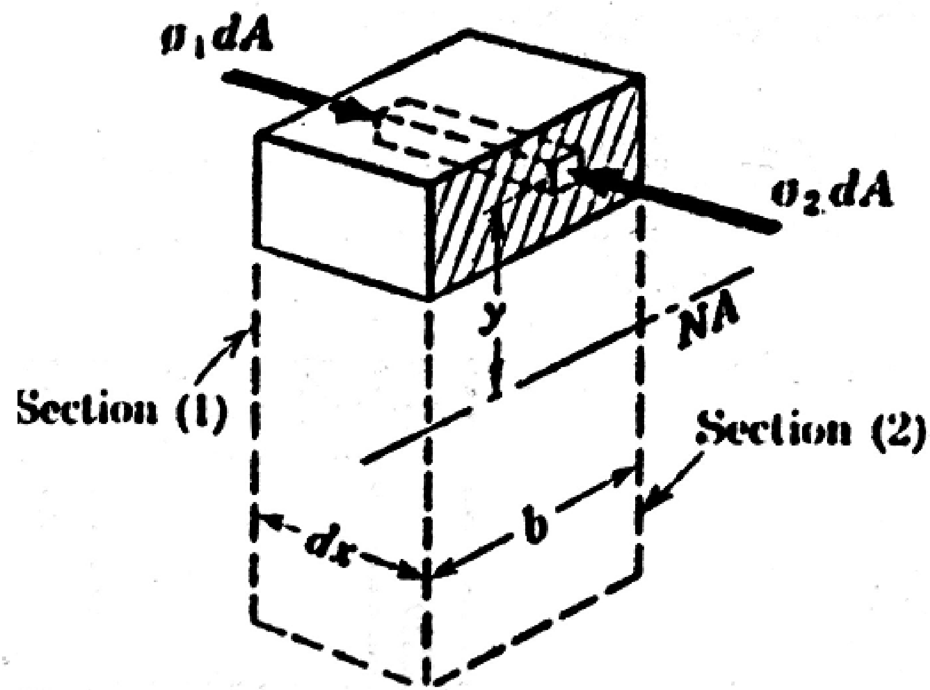


Fig : 2

Horizontal shear stress of beams

But, $dF = \tau b\, dx$, where,

τ - Average horizontal shear stress

$M_2 - M_1$ = Differential change in bending moment,

$= dM$

$$\therefore \quad dF = \frac{dM}{I b\, dx} \int_{y_1}^{c} y\, dA$$

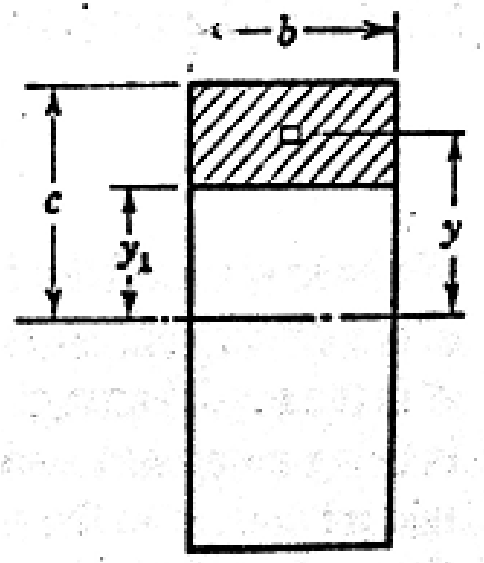
But, $\dfrac{dM}{dx} = V$, the vertical shear

Horizontal shear stress of beams

$$\therefore \tau = \frac{V}{Ib}\int_{y_1}^{c} y\, dA$$

$$= \frac{V}{Ib} A' \bar{y}$$

$$\therefore \boxed{\tau = \frac{VQ}{Ib}}$$

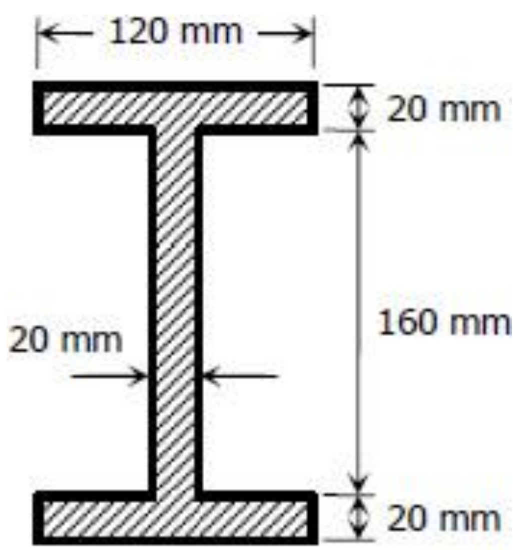


- Here, A’ is the shaded area shown in fig.
  $\bar{y}$ is the moment arm of the area A’ with respect to neutral axis.

- Horizontal shear stress is maximum at neutral surface.
- Horizontal shear stress is zero at top and bottom surfaces.

Problem# 575 (singer)

- Determine the maximum and minimum shearing stress in the web of the wide flange section in Fig. if V = 100 kN.



Problem# 588 (singer)

- The distributed load shown in Fig. is supported by a wide-flange section of the given dimensions. Determine the maximum value of w_o that will not exceed a flexural stress of 10 MPa or a shearing stress of 1.0 MPa.